



# Introduction

It is Thursday, the 23<sup>rd</sup> of September, 2010.

Just three months ago you graduated from the GUC – after having received excellent grades in your Majors “Economics” 😊 and “Finance”.

Since two months you are now working as a contract consultant in the loan department of the Pharaoh Bank.

You found the job quite interesting so far.

However, there were no *real* challenges that could prove your *outstanding* skills in contract design.

But this will change in a few seconds...



# Introduction

Just 15 minutes before you are going to leave for the weekend, your superior (the head of the loan department) arrives.

*Boss:* “Forget the weekend, there is an emergency case.”

*You:* “But, Boss, I planned this nice trip to Luxor...”

*Boss:* “*Eh?* Those professors at the GUC should have made clear that the humorless Germans are world champions in exports – mostly because of one thing: They show *real* dedication to their jobs.

And I am sure there are other GUCians who understood this lesson better than you did and who are eager to join our department when I am going to offer them *your* position...”



### Introduction

*You:* “Ok. Ok. What is the problem?”

*Boss:* “There is this German investor – Michael Luftmacher. He is asking us to provide the loan for his new project: The Salahedin Business Centre at the Citadel. It seems to be a very sound project, but ...

*You:* “But?”

*Boss:* “One week ago, one of my friends accidentally overheard Michael at the “Giza Golf Club” boasting he had just discovered a way how to make a fortune at the Roulette tables in Las Vegas. We checked his claim and found out that the probability for this “plan” to be successful is just 60 %!

Thus, on the one hand we are very eager to get into business with this guy. On the other hand... How can we make sure that Michael will use the money for his business project and will not engage in a gambling mission?”



### Introduction

*You:* “This setting reminds me of a case study we once investigated at the GUC about a thing called “Moral Hazard”. It is highly theoretical and I always assumed this conception has no value in real life. But perhaps I should reconsider ...”

*Boss:* “Perfect. Would do you need?”

*You:* “Definitely all the data we can get concerning Michael’s company and the project alternatives. I then will set up a game to analyze his decision options ...”

*Boss:* “A what? Game? What are you talking about? What games got to do with our problem?”



# Introduction

*You:* “Do not worry, boss. I will give you a very short introduction to Game Theory – it is a mathematical tool that helps you to identify possible solutions to our problem.”

*Boss:* “O.k. But do not overdo it – you know I do not like Math. I just need to understand the main idea. Be ready to present your analysis next Monday, 8 AM at my office.

By the way – this project is very important to us. If it works out well, we can apply **our** solution to a lot of similar cases. If it does not, then do not be surprised to find out that **your** solution did cost **you** the job.”



# Introduction

*Monday – 8 AM*

*You:*

“Good morning, ladies and gentlemen.

Let me first illustrate why moral hazard is a problem from our perspective as a potential lender in business operations...



### Simplification by Assumption

*You:* “In order to highlight the problem of moral hazard, let me assume that...”

... the market interest rate is 0 %.

Reason for this simplification:

- We all know the implications of the time value of money – right?
- Thus, there is no need to complicate the investigation with something that is momentarily *not* in the analysis' focus...



### The Hazard in Moral Hazard

Once upon a time in Taxfreezonia, there was a company completely owned by Mr. Smart.

This company was going to be liquidated for sure just after one period ( $= t_1$ ).

At  $t_1$  the company was supposed to repay its outstanding debt of 100 MU (MU = Money units).

At  $t_0$  it had accumulated a profit of 100 MU.

What to do with the profit in  $t_0$ ?





## The Hazard in Moral Hazard

What to do with the profit of 100 MU in  $t_0$ ?

Option 1: Play "Safe"

- Mr. Smart takes out 100 MU as dividend in  $t_0$ .
- Continuation of normal business operations (*Clarification*: These operations do not require the 100 MU).

Option 2: Play "Hazard"

- Mr. Smart takes out only 70 MU as dividend in  $t_0$ .
- The normal business operations are continued.
- Mr. Smart invests the remaining 30 MU in an additional risky project.



### The Hazard in Moral Hazard

Both the normal operations and the additional project are exposed to risk:

- “Boom times” with probability of  $p = 0.5$
- “Bust times” with probability of  $1-p = 0.5$

Depending on the option chosen by Mr. Smart in  $t_0$  as well as depending on the state of nature (“Boom” or “bust”), the company will generate different end values in  $t_1$ .

These end values are to be distributed among Mr. Smart and the debt holders.

Keep in mind, debt contracts imply that debt holders represent “first-priority-claimants” on a company’s assets (here: end value).



## The Hazard in Moral Hazard

Strategy	State of Nature	
	Boom (p = 0.5)	Bust (1-p = 0.5)
End value when Smart chooses "Safe"	110 MU	70 MU
End value when Smart chooses "Hazard"	200 MU	5 MU

Company's total value when Mr. Smart chooses "Safe":

$$TV_S = 100 + 0.5 \cdot 110 + 0.5 \cdot 70 = 190$$

Dividend in  $t_0$      $E(\text{Boom end value in } t_1)$      $E(\text{Bust end value in } t_1)$

Company's total value when Mr. Smart chooses "Hazard":

$$TV_H = 70 + 0.5 \cdot 200 + 0.5 \cdot 5 = 172.5$$

Dividend in  $t_0$      $E(\text{Boom end value in } t_1)$      $E(\text{Bust end value in } t_1)$

Because  $TV_S > TV_H$ , the additional project's NPV is negative and it should be rejected.

*Recommendation:*

Mr. Smart should play safe to maximize total value.



## The Hazard in Moral Hazard

Strategy	State of Nature	
	Boom (p = 0.5)	Bust (1-p = 0.5)
End value when Smart chooses "Safe"	110 MU	70 MU
End value when Smart chooses "Hazard"	200 MU	5 MU

But Mr. Smart is not interested in maximizing total value!

He is only interested in maximizing his own wealth.

Mr. Smart's wealth (= W) when he chooses "Safe":

$$W_S = \underbrace{100}_{\text{Dividend in } t_0} + 0.5 \cdot \left( \underbrace{110}_{\text{Full debt repayment}} - \underbrace{100}_{\text{Full debt repayment}} \right) + 0.5 \cdot \left( \underbrace{70}_{\text{Residual debt repayment}} - \underbrace{70}_{\text{Residual debt repayment}} \right) = 105$$

Mr. Smart's wealth when he chooses "Hazard":

$$W_H = \underbrace{70}_{\text{Dividend in } t_0} + 0.5 \cdot (200 - 100) + 0.5 \cdot (5 - 5) = 120$$

Because  $W_H > W_S$ , Mr. Smart prefers playing "Hazard".



## The Hazard in Moral Hazard

Strategy	State of Nature	
	Boom ( $p = 0.5$ )	Bust ( $1-p = 0.5$ )
End value when Smart chooses "Safe"	110 MU	70 MU
End value when Smart chooses "Hazard"	200 MU	5 MU

By choosing "Hazard", Mr. Smart can improve his wealth by:

$$+\Delta W = 120 - 105 = 15$$

The debt holders lose by this shirking strategy:

$$-\Delta V_D = 0.5 \cdot (100 + 70) - 0.5 \cdot (100 + 5) = 85 - 52.5 = 32.5$$

Total loss:

$$32.5 - 15 = 17.5$$



# The Hazard in Moral Hazard

### *Moral hazard:*

Mr. Smart (as sole equity holder) can improve his wealth position at the expense of debt holders by deviating to risky, total-wealth-destroying projects.

This problem will show up, when Mr. Smart is able to hide his “hazard”-strategy from being monitored by the bond holders (randomness necessary to mask his actions).

### *Conclusion:*

Moral hazard in the investigated setting requires asymmetric information between bond holders (outsiders) and equity holders (insiders) concerning project choices.



# Game Theory – A Very Short Overview

*Boss:* “That part of the presentation was not too bad. Actually it was a nice illustration for a problem my guts told me that it is highly relevant to our business.

But how can we solve a problem of moral hazard?”

*You:* “Boss, that is where Game Theory comes into play. Let me provide a short introduction to this widely used tool of analysis.

Game Theory investigates situation of **strategic interdependence** – the actions of one actor influence not only her / his well being, but the well being of other individuals, too.

Thus, the decision to take an umbrella with you (rain protection when it rains, only a burden when the sun shines) is **not** a situation of strategic interdependence, because there are no human “opponents” – it is just you and nature.”



# Game Theory – a Very Short Overview

Game Theory uses the following “ingredients” to analyze situations of strategic interdependence.

**Players:** Who must be considered a participant because of her / his ability to influence the outcome?

**Strategies:** What actions are available to each player?

**Outcomes:** What results are generated when each player chooses a specific strategy?

**Equilibria:** If all players employ their optimal strategies in accordance to the *rules of the game*, what will be the value / solution of the game?





# Game Theory – a Very Short Overview

Let me illustrate how Game Theory works by introducing the so-called “Prisoner’s dilemma”.

This game has become famous for several reasons:

- (1) It’s solution is pretty easy to grasp.
- (2) It can be applied to a vast array of different situations.
- (3) In the Prisoner’s Dilemma all guns are loaded against **cooperation**.  
Thus, it can serve as a benchmark – id est, it helps us to understand the conditions to be met in order to successfully engage in cooperations in real life.



## The Prisoner's Dilemma

The background story to this game goes like this ...

Two criminals committed a serious crime and are now interrogated in *different* cells by the police.

However, the police has only *clear* evidence for a minor offense: The criminals were caught carrying guns.



# The Prisoner's Dilemma

Thus, the police proposes to **each** criminal the following “deal”:

If **both** of you continue to **deny** that you and your partner committed the serious crime, you **both** receive a sentence of 1 year for carrying guns.

If **one** of you **confesses** while the **other** continues to **deny**, then the **confessor** is set free immediately and the **denier** is sentenced to the maximum punishment: 11 years in jail.

If **both** of you **confess**, then – because of your willingness to help the police solving a crime – **each of you** will have **lowered** his sentence to 9 years.

Keep in mind:  
You can **not** be sure what your partner is going to do during the interrogation.  
Furthermore, there is **no way** to punish/reward him after you found out.



# The Prisoner's Dilemma

		Criminal 2 = Column player	
		confess	deny
Criminal 1 = Row player	confess	(9,9)	(0,11)
	deny	(11,0)	(1,1)

Preference for both criminals:  
**Minimize** the number of years in prison!  
Criminal 2 receives the highest penalty.

Both criminals will only be punished for carrying guns  
What strategy is optimal to each prisoner?

Strategies available to Criminal 1

Both criminals receive a sentence of 9 years because both had chosen independently to confess.  
**Blue:** Payoff [in years] for Criminal 1;  
**Red:** Payoff [in years] for Criminal 2.



## The Prisoner's Dilemma

		Criminal 2 = Column player	
		confess	deny
Criminal 1 = Row player	confess	(9, 9)	(0, 11)
	deny	(11, 0)	(1, 1)

To solve the game, let us first put ourselves into the shoes of Criminal 1.

Regardless of Criminal 2's choice, Criminal 1 is **always** better off when playing "confess".

### Reason:

When Criminal 2 plays "confess", Criminal 1 is better off, when playing himself "confess" instead of "deny": 9 years < 11 years.

When Criminal 2 plays "deny", Criminal 1 is better off, when playing himself "confess" instead of "deny": 0 years < 1 year.

Because this is a **symmetric game**, the same line of reasoning applies to Criminal 2:

He will **always** choose confess, too.



## The Prisoner's Dilemma

		Criminal 2 = Column player	
		confess	deny
Criminal 1 = Row player	confess	(9,9)	(10,0)
	deny	(0,10)	(1,1)

In other words:  
The strategy “confess” **strictly dominates** the strategy “deny” for both players.

Consequently, we can remove the strategy “deny” as a relevant option for both players.

The **equilibrium** in dominating strategies consists in both players choosing “confess”.



## Cournot–Nash Equilibrium

Unfortunately, the vast majority of game constellations features *no* dominating strategies.

Therefore, we need a different conception of equilibrium to handle these cases.



# Cournot–Nash Equilibrium

(Cournot-)Nash Equilibrium:

1. Assume that all players except player  $i$  have already chosen their **optimal** strategies.
2. Now have a look at each strategy available to player  $i$ .
3. Check for each of these strategies  $s_i$ : Under the assumption that all other players stick to their optimal strategies (rationality!), will **player  $i$**  stick to  $s_i$  or does she / he has an incentive to deviate to another strategy?
4. When there is **no incentive** to switch to another strategy, then strategy  $s_i$  is a possible **candidate** for the Nash Equilibrium.
5. Do this investigation for all strategies and players.
6. Those strategy combinations identified as candidates that actually represent the **best response to each other** (all players have no incentive to deviate), constitute a Nash equilibrium.





## Cournot–Nash Equilibrium: Example

The payoffs represent arbitrarily chosen utility values. We assume that each player seeks to **maximize** only his / her own **utility**.

		Column player	
		call	wait
Row player	call	(0,0)	(3,5)
	wait	(5,3)	(1,1)

*Note: In the original image, pink arrows point from the (0,0) cell to (3,5), from (5,3) to (1,1), and from (1,1) to (3,5). This indicates that for the Column player, 'wait' is a dominant strategy, and for the Row player, 'wait' is a dominant strategy.*

Assume, you have a telephone conversation with your friend.

Suddenly the connection breaks down.

**Who** should **call** again and who should **wait**?

One of you calls while the other waits.  
Both of you benefit from the conversation, but the caller has to pay for making the call...



## How to Identify Cournot–Nash Equilibria

There is **no** equilibrium in **dominating** strategies.  
Can you show this?

		Column player	
		call	wait
Row player	call		(3,5)
	wait	(5,3)	(1,1)

We focus on the “Row player”.

In regard to the “Column player” we assume (for the moment) that “call” represents her / his optimal strategy.

Then we can conclude that “call” does **not** represent the **optimal response** by “Row player” because she / he has an incentive to **deviate** to “wait”:  $5 > 0$ .

### Implications:

The strategy combination “call-call” **does not** represent a Nash equilibrium.

The strategy combination “call by Column player – wait by Row player” is a **candidate** for a Nash equilibrium.



## How to Identify Cournot–Nash Equilibria

In fact, this combination is *not only* a candidate – it *is* a Nash Equilibrium!

		Column player	
		call	wait
Row player	call		(3,5)
	wait	(5,3)	(1,1)

*Reason:*

When Row player chooses “wait” as her / his optimal strategy, Column player has *no* incentive to deviate from “call” to “wait”:  $3 > 1$ .

Because of the **symmetry** in the payoffs to both players, the strategy combination “call” by Row player and “wait” by column player represents a Nash equilibrium, too.



## How to Identify Cournot–Nash Equilibria

What about the combination “wait”-“wait”?

		Column player	
		call	wait
Row player	call	(0,0)	(3,5)
	wait	(5,3)	(1,1)

Diagram details: The (3,5) cell is circled in grey. The (5,3) cell is circled in grey. A pink arrow points from (3,5) to (5,3). A blue arrow points from (1,1) to the right. A red arrow points from below to (1,1).

“Wait”-“wait” can *not* be a Nash Equilibrium.

*Reasons:*

When Row player chooses “wait” as her / his optimal strategy, Column player has the incentive to deviate from “wait” to “call”:  $3 > 1$ .

When Column player chooses “wait” as her / his optimal strategy, Row player has an incentive to deviate from “wait” to “call”:  $3 > 1$ .

Nice to know:

This game *lacking one unique equilibrium* illustrates why in real life we have **cultural conventions**: “The person who called first, should be the caller after the break-down.”



# Cournot–Nash Equilibria: Conclusions

Nash Equilibria represent solutions to games where no strictly dominating strategies are available.

Because **no player** has an incentive to deviate from her / his strategy in a Nash equilibrium, Nash Equilibria are **self-enforcing**.

The Nash equilibrium **lowers** the conditions to be met for strategy combinations to represent an equilibrium.

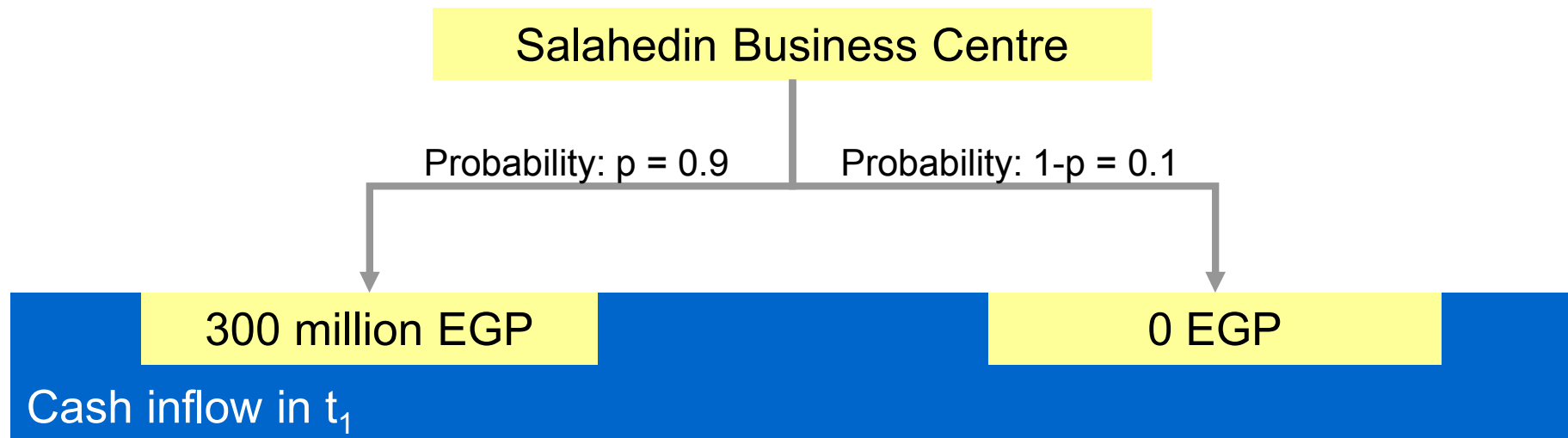
**Explanation:** The corresponding strategies represent **only** the best responses to **each** other, **not** the best response to **all** possible strategy combinations!

Consequently, applying the Nash equilibrium comes with a price: The probability that a game features **no unique** equilibrium in pure strategies ( $\Leftrightarrow$  mixed strategies) is pretty high.



## Michael Luftmacher and the Pharaoh Bank

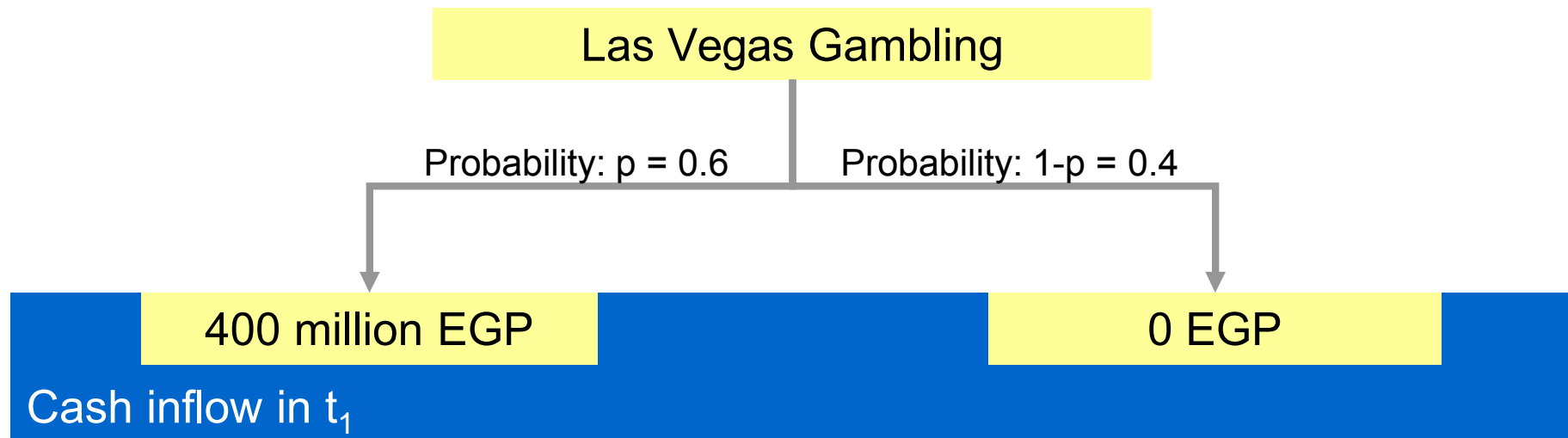
Michael Luftmacher needs **100 million EGP** to finance his “Salahedin business centre” project in  $t_0$ .





## Michael Luftmacher and the Pharaoh Bank

Michael Luftmacher could also use the **100 million EGP** to finance his “Las Vegas gambling mission” in  $t_0$ .





# Michael Luftmacher and the Pharaoh Bank

The moral hazard: The Pharaoh Bank is not able to observe Michael's choice of project in  $t_0$  ("**hidden action**").

**Randomness** to mask Michael's choice:

Assume, Michael promises to go for the Salahedin project, but actually chooses Las Vegas.

If he then ends up with 0 EGP in  $t_1$ , he can always claim that he had chosen the Salahedin project, but it went wrong (Probability: 10 %).





# Michael Luftmacher and the Pharaoh Bank

Both parties are *risk-neutral*.

Both parties operate with a *risk-free interest rate* of 10 % to reflect the time value of money.

The Pharaoh Bank is *not* interested in making a profit: This contract may serve as a blueprint to future operations. Thus, a *Zero profit* (on average) to the bank is sufficient.

The Bank is aware that Michael has a nice villa at the Côte-d'Azur. Its worth: 20.02 Million EGP in  $t_1$ .



## Michael Luftmacher and the Pharaoh Bank

The “obvious” strategies available to the bank in  $t_0$  :

Offer the 100 Million EGP *unsecured* to the conditions required by the *Salahedin* data.

Offer the 100 Million EGP *unsecured* to the conditions required by the *Vegas* data.



## Michael Luftmacher and the Pharaoh Bank

Offering the 100 Million EGP *unsecured* (= u) to the conditions required by the *Salahedin* (= S) data, implies:

$$100 = \frac{E(FV)}{1.1} = \frac{0.9 \cdot FV + 0.1 \cdot 0}{1.1} = \frac{0.9 \cdot (100 \cdot (1 + r_S^u)) + 0.1 \cdot 0}{1.1}$$

Solving this equation results in:

Present value of loan offered to Michael equals  $\frac{0.9 \cdot (100 \cdot (1 + r_S^u)) + 0.1 \cdot 0}{1.1}$  present value of **expected** debt repayment (= FV).

must reflect the fact that the future value of the 100 million EGP will only be repaid with a probability of 90 %.

$$\Leftrightarrow \frac{1.1}{0.9} - 1 = r_S^u \approx 22.22\%$$



# Michael Luftmacher and the Pharaoh Bank

Offering the 100 Million EGP *unsecured* (= u) to the conditions required by the *Vegas* (= V) data, implies:

$$100 = \frac{E(FV)}{1.1} = \frac{0.6 \cdot FV + 0.4 \cdot 0}{1.1} = \frac{0.6 \cdot (100 \cdot (1 + r_V^u)) + 0.4 \cdot 0}{1.1}$$

Solving this equation results in:

$$100 = \frac{0.6 \cdot (100 \cdot (1 + r_V^u)) + 0.4 \cdot 0}{1.1} \Leftrightarrow 100 \cdot 1.1 = 0.6 \cdot (100 \cdot (1 + r_V^u))$$

$$\Leftrightarrow \frac{1.1}{0.6} - 1 = r_V^u \approx 83.33\%$$



# Michael Luftmacher and the Pharaoh Bank

		M. Luftmacher = ML	
		choose Salahedin	choose Vegas
Pharaoh Bank = PB	unsecured Salahedin loan r = 22 %	(0, 145.46)	
	unsecured Vegas loan r = 83.33 %		

$$\Pi_{PB} = -100 + 0.9 \cdot \frac{122.22}{1.1} \approx 0$$

$$\Pi_{ML} = +100 - 100 + 0.9 \cdot \frac{300 - 122.22}{1.1} \approx 145.46$$



# Michael Luftmacher and the Pharaoh Bank

		M. Luftmacher = ML	
		choose Salahedin	choose Vegas
Pharaoh Bank = PB	unsecured Salahedin loan r = 22 %	(0, 145.46)	(-33.33, 151.52)
	unsecured Vegas loan r = 83.33 %		

$$\Pi_{PB} = -100 + 0.6 \cdot \frac{122.22}{1.1} \approx -33.33$$

$$\Pi_{ML} = +100 - 100 + 0.6 \cdot \frac{400 - 122.22}{1.1} \approx 151.52$$



# Michael Luftmacher and the Pharaoh Bank

		M. Luftmacher = ML	
		choose Salahedin	choose Vegas
Pharaoh Bank = PB	unsecured Salahedin loan r = 22 %	(0, 145.46)	(-33.33, 151.52)
	unsecured Vegas loan r = 83.33 %	(50, 95.96)	

$$\Pi_{PB} = -100 + 0.6 \cdot \frac{183.33}{1.1} \approx 50$$

$$\Pi_{ML} = +100 - 100 + 0.9 \cdot \frac{300 - 183.33}{1.1} \approx 95.96$$



## Michael Luftmacher and the Pharaoh Bank

		M. Luftmacher = ML	
		choose Salahedin	choose Vegas
Pharaoh Bank = PB	unsecured Salahedin loan $r = 22\%$	(0, 145.46)	(-33.33, 151.52)
	unsecured Vegas loan $r = 83.33\%$	(50, 95.96)	(0, 118.18)

$$\Pi_{PB} = -100 + 0.6 \cdot \frac{183.33}{1.1} \approx 0$$

$$\Pi_{ML} = +100 - 100 + 0.6 \cdot \frac{400 - 183.33}{1.1} \approx 118.18$$





## Michael Luftmacher and the Pharaoh Bank

What is the **equilibrium** in this game?

		M. Luftmacher = ML	
		choose Salahedin	choose Vegas
Pharaoh Bank = PB	[Redacted]		
	unsecured Vegas loan $r = 83.33\%$	(50, 95.96)	(0, 118.18)

PB has the **dominating strategy** to offer the Vegas loan, because ...

$50 > 0$  and  $0 > -33.33$ .

ML reacts by choosing the Vegas project, because  $118.18 > 95.96$ .



## Michael Luftmacher and the Pharaoh Bank

You:

“Boss, there is a major problem with this equilibrium.”

		M. Luftmacher = ML	
		choose Salahedin	choose Vegas
Pharaoh Bank = PB	unsecured Salahedin loan r = 22 %	(0, 145.46)	(-33.33, 151.52)
	unsecured Vegas loan r = 83.33 %	(50, 95.96)	(0, 118.18)

In comparison to this equilibrium, ML himself would **prefer** to get the Salahedin loan conditions and **really** choose the Salahedin project. Reason:  $145.45 > 118.18$ .

Consequently, the challenge is do introduce a **contract element** that makes sure that ML is **discouraged** from choosing “Vegas” when offered the Salahedin loan.

Nice to know:

Moral hazard / adverse selection very often **backfires** to the agent, because she / he is not able to **credibly** convince the principal that he will behave “nicely” / owns the good quality.



## Michael Luftmacher and the Pharaoh Bank

To overcome this problem, we have to make sure that – when offered the **Salahedin loan** – ...

		M. Luftmacher = ML	
		choose Salahedin	choose Vegas
Pharaoh Bank = PB	unsecured Salahedin loan r = 22 %	(0, 145.46)	(-33.33, 151.52)
	unsecured Vegas loan r = 83.33 %	(50, 95.96)	(0, 118.18)

... ML's expected profit from **choosing Vegas** ...

... is **lower than** (or at least equal to)...

... his **profit** from **choosing Salahedin**.

To achieve this, we introduce a **collateral** that lowers ML's profit in the case of being **unable to repay** his outstanding debt.

Because the **probability of bankruptcy** is much **higher** in the case of the **Vegas project**, the introduction of a collateral makes the Vegas choice **particularly unattractive**.



## Michael Luftmacher and the Pharaoh Bank

To determine the amount necessary as a collateral, we have to meet *two conditions*.

		M. Luftmacher = ML	
		choose Salahedin	choose Vegas
Pharaoh Bank = PB	unsecured Salahedin loan $r = 22\%$	(0, <span style="color: red;">145.46</span> )	(-33.33, <span style="color: red;">151.52</span> )
	unsecured Vegas loan $r = 83.33\%$	(50, <span style="color: red;">95.96</span> )	(0, <span style="color: red;">118.18</span> )

*Reminder:*  
In the case of bankruptcy, the collateral goes to the Pharaoh Bank!

**Condition 1:** The collateral (= C) should force ML to choose the Salahedin project. Thus, the Zero profit condition for the Pharaoh Bank changes to:

$$\begin{aligned} \Pi_{PB} &= -100 + 0.9 \cdot \frac{100 \cdot (1+r_S^C)}{1.1} + 0.1 \cdot \frac{C}{1.1} = 0 \Leftrightarrow 100 - \frac{0.1 \cdot C}{1.1} = 0.9 \cdot \frac{100 \cdot (1+r_S^C)}{1.1} \\ &\Leftrightarrow \frac{1.1}{0.9} - \frac{0.1 \cdot C}{0.9 \cdot 100} = 1 + r_S^C \Leftrightarrow \frac{1.1}{0.9} - \frac{0.1 \cdot C}{90} - 1 = r_S^C \Leftrightarrow r_S^C = 0.2222 - 0.001111 \cdot C \end{aligned}$$



## Michael Luftmacher and the Pharaoh Bank

We can plug in the result of condition 1 into condition 2 ...

		M. Luftmacher = ML	
		choose Salahedin	choose Vegas
Pharaoh Bank = PB	unsecured Salahedin loan r = 22 %	(0, 145.46)	(-33.33, 151.52)
	unsecured Vegas loan r = 83.33 %	(50, 95.96)	(0, 118.18)

*Reminder:*  
In the case of bankruptcy (40%), ML **loses** the collateral!

**Condition 2:** To make sure that ML actually chooses Salahadin, the collateral must reduce the profit from choosing Vegas at least to 45.45 million EGP.

$$\Pi_{ML}^V = 0.6 \cdot \frac{400 - 100 \cdot (1 + 0.2222 - 0.001111 \cdot C)}{1.1} - 0.4 \cdot \frac{C}{1.1} = 145.45 \Leftrightarrow C \approx 20.02$$

$= r_S^C$



# Michael Luftmacher and the Pharaoh Bank

Fortunately, ML is able to offer the villa (worth in  $t_1$ : 20.02 Million EGP) as a collateral [What a coincidence! 😊 Required collateral: 20.02 Million EGP = villa value].

With this collateral the risk-adjusted interest rate for PB changes to:

$$r_s^C = 22.22\% - 0.001111 \cdot 20.02 \approx 20\%$$

Financing the Salahedin project results in the following profit for PB:

$$-100 + 0.9 \cdot \frac{120}{1.1} + 0.1 \cdot \frac{20.02}{1.1} \approx 0$$

Financing the Vegas project results in the following profit for PB:

$$-100 + 0.6 \cdot \frac{120}{1.1} + 0.4 \cdot \frac{20.02}{1.1} \approx -27.27$$

Choosing the Salahedin project, ML achieves a profit of:

$$+100 - 100 + 0.9 \cdot \frac{300-120}{1.1} - 0.1 \cdot \frac{20.02}{1.1} \approx 145.45$$

Choosing the Vegas project, ML achieves a profit of:

$$-100 + 100 + 0.6 \cdot \frac{400-120}{1.1} - 0.4 \cdot \frac{20.02}{1.1} \approx 145.45$$



## Michael Luftmacher and the Pharaoh Bank

Thus, when using the collateral to secure the Salahedin project, the payoffs change to ...

		M. Luftmacher = ML	
		choose Salahedin	choose Vegas
Pharaoh Bank = PB	secured Salahedin loan $r = 20\%$ , $C = 20.02$	(0, 145.46)	(-27.27, 145.45)
	unsecured Vegas loan $r = 83.33\%$	(50, 95.96)	(0, 118.18)

Because ML is *indifferent* between choosing Salahedin and Vegas when offered the **secured loan**, PB can induce ML – by increasing the collateral amount by an infinite small amount – to **prefer** the Salahedin project.

Due to this (easy to establish) preference for actually choosing Salahedin by ML, we now have a second Nash equilibrium.